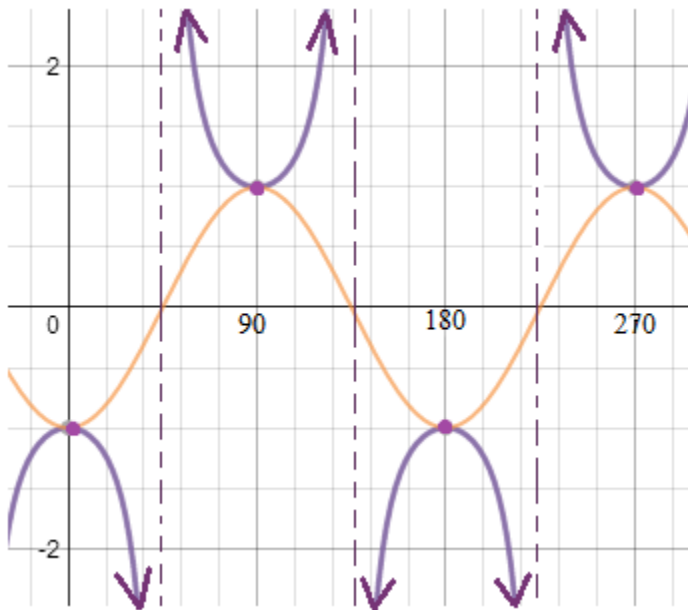
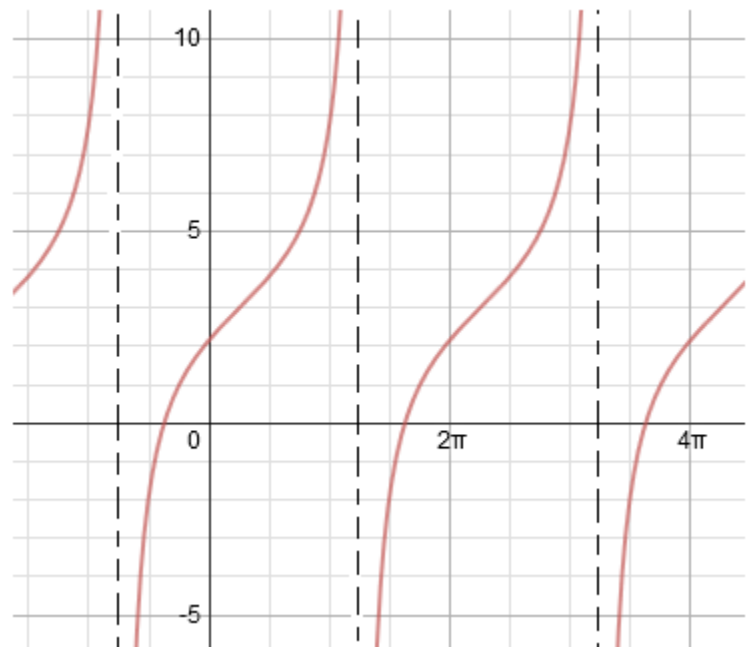


Introduction to Periodic Trig Functions 2:

Tangent Graphs

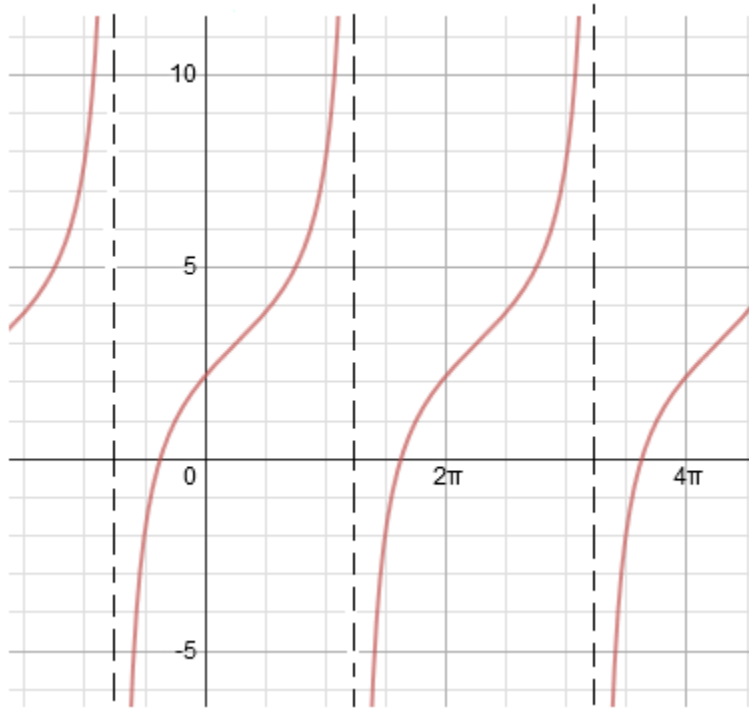


and,
Reciprocal Sketches

Notes/examples of trig values and the components of trig graphs

Includes 2 practice tests (and solutions)

Tangent Functions



Topics include asymptotes, period, amplitude, horizontal and vertical shifts, reflection, and more.

Sketching the Parent Function:

$y = \tan(x)$
(radians)

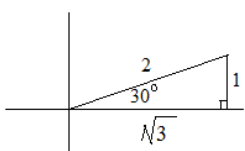
$y = \tan \Theta$
(degrees)

Tangent Functions

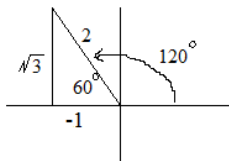
Tangent = $\frac{\text{opposite}}{\text{adjacent}}$

$\rightarrow \frac{y}{x}$

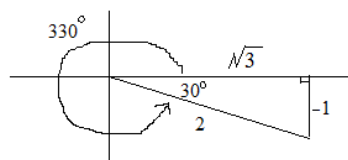
A few examples of common angles:



$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\tan 120^\circ = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$$

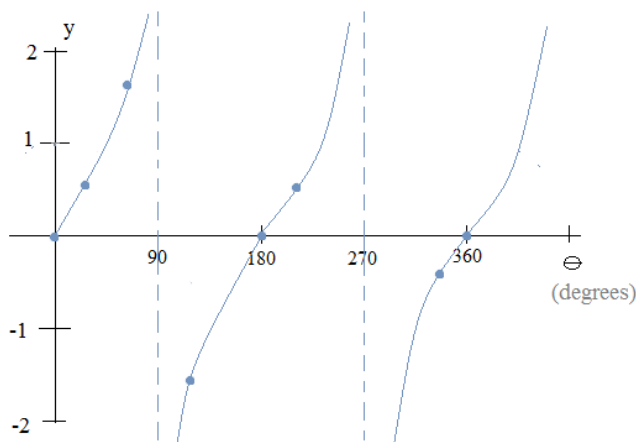


$$\tan 330^\circ = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

The following is a table of chosen values:

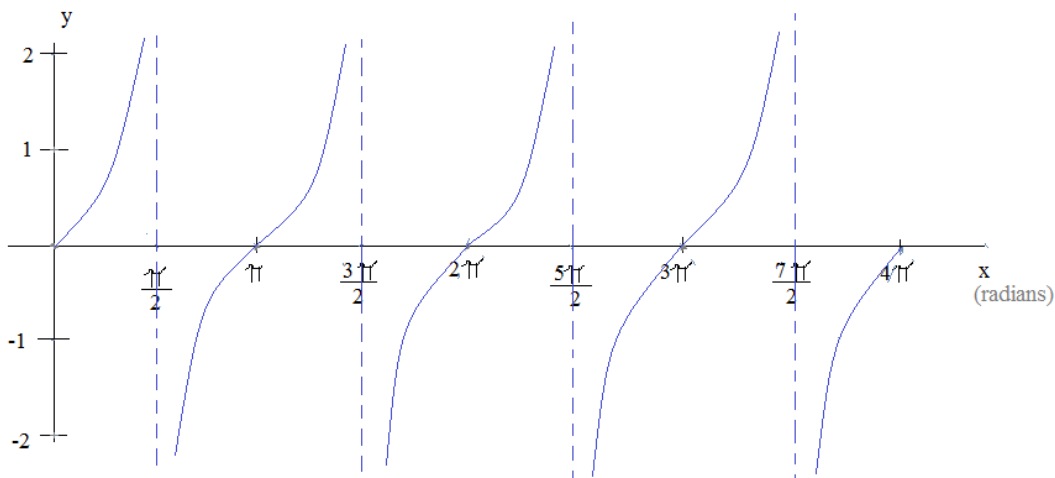
Θ	0	30	60	90	120	180	210	270	330	360
$y = \tan \Theta$	0	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{1}{0}$	$-\sqrt{3}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{0}$	$\frac{1}{\sqrt{3}}$	0
				Undefined				Undefined		

Then, plot the points...



It's a periodic function -- it will repeat the pattern of y-values at a regular interval of 180° . (The period is 180°)

$y = \tan x$



A cycle is a complete pattern repetition. This sketch contains 4 cycles.

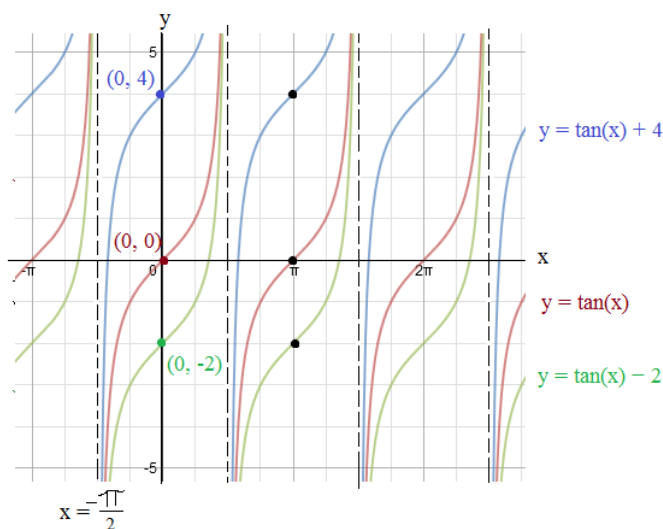
The period is π (Horizontal length of one cycle)

Tangent Functions: 4 components

The four parts of the tangent function can stretch, shift, reflect, and compress the parent function (graph).

$y = \tan(x)$ is the parent function

Vertical Shift:



$$y = A \tan(B(x + C)) + D$$

A: Amplitude (magnitude)

B: Period (cycles/ π)

C: Horizontal Shift

D: Vertical Shift

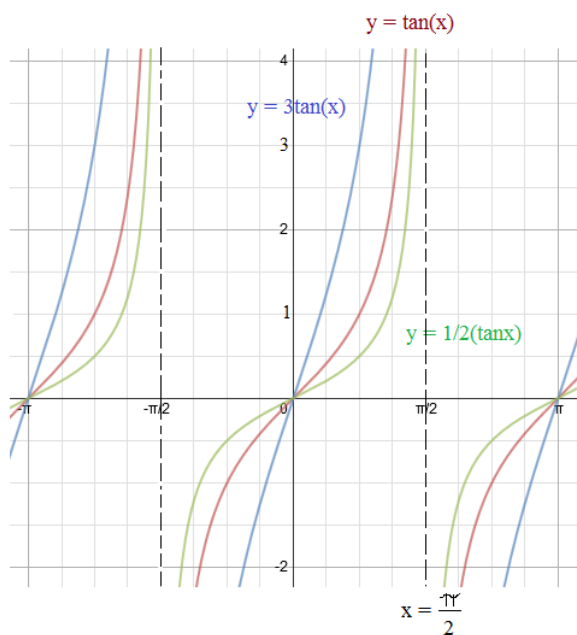
$$\text{If } x = \pi$$

$$y = \tan(x) \rightarrow \tan(\pi) = 0$$

$$y = \tan(x) + 4 \rightarrow \tan(\pi) + 4 = 4$$

$$y = \tan(x) - 2 \rightarrow \tan(\pi) - 2 = -2$$

Amplitude:

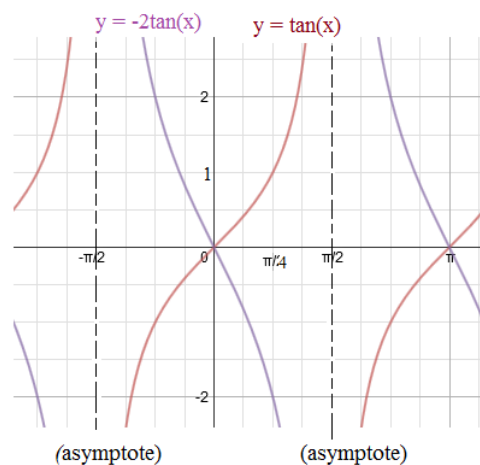


x	$\tan x$	$3 \tan x$	$\frac{1}{2} \tan x$	$-2 \tan x$
0	0	0	0	0
$\frac{\pi}{4}$	1	3	$\frac{1}{2}$	-2
$\frac{\pi}{2}$	undefined	undefined	undefined	undefined
π	0	0	0	0
$\frac{3\pi}{4}$	-1	-3	$-\frac{1}{2}$	2

When the coefficient gets larger, the tangent curves *stretch*...

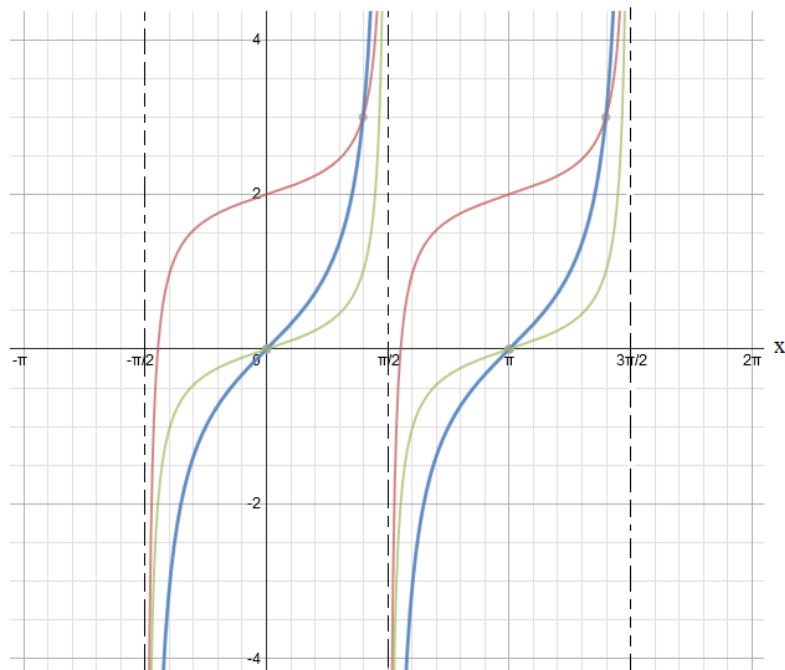
When the coefficient is negative, the output is reflected over the x-axis.

NOTE: The amplitude (A) and vertical shift (D) are numbers *outside the function*. So, they affect changes that are *up and down*.



Tangent Functions: Amplitude and Vertical Shift Illustrations

I. Sketch two cycles of the function $f(x) = \frac{1}{3} \tan x + 2$



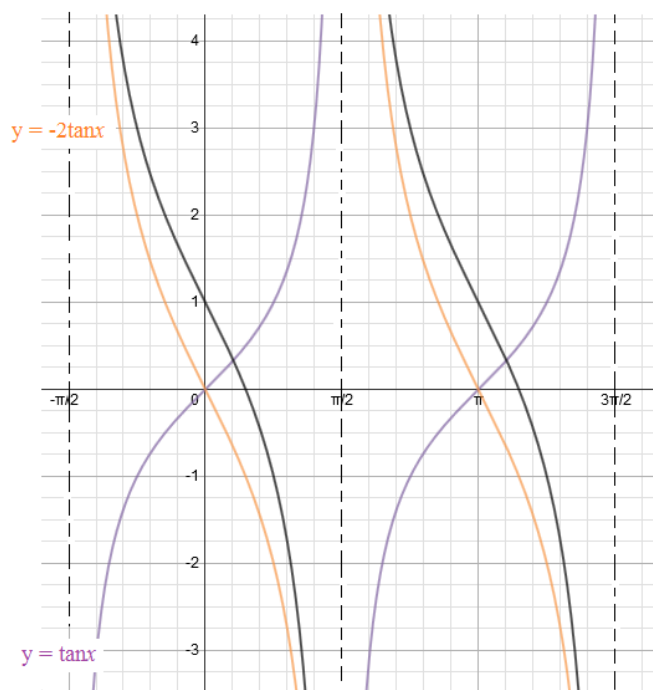
1) Sketch the parent function $y = \tan x$

2) Decrease ("shrink") the amplitude by a factor of 3 $y = \frac{1}{3} \tan x$

3) Shift the graph up 2 units $y = \frac{1}{3} \tan x + 2$

Note: The asymptotes provide a good outline for your sketch...

II. Sketch $y = -2 \tan x + 1$



1) Parent Function $y = \tan x$

2) "Stretch" and "Reflect" $y = -2 \tan x$

3) Vertical Shift $y = -2 \tan x + 1$

Notice, the "new middle" is at $y = 1$
(instead of the x-axis)

And, because the (A) value is 'negative',
the curve gets 'flipped'/'reflected'
(i.e. goes from upper left to lower right)

Tangent Functions: 4 components

The four parts of the tangent function can stretch, shift, reflect, and compress the parent function (graph).

Since the A and D terms are outside the function, the changes affect the vertical components..
(vertical "stretch", reflection, and shift)

Since the B and C terms are inside the function, they will affect the horizontal shape of the graph....

$$y = A \tan B(x + C) + D$$

A: Amplitude (magnitude)
B: Period (cycles/ π)
C: Horizontal Shift
D: Vertical Shift

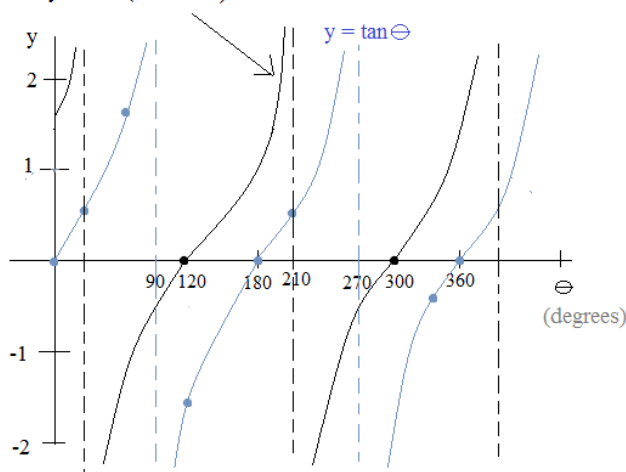
Horizontal ("Phase") Shift:

Question: If $\tan 180^\circ = 0$, then where does $\tan(\Theta + 60^\circ) = 0$?

Answer: $\Theta = 120^\circ$, because $\tan(120 + 60) = 0$.

Implication: $180^\circ \Rightarrow 120^\circ$ (shift 60° to the left)

Example 1: $y = \tan(\Theta + 60^\circ)$



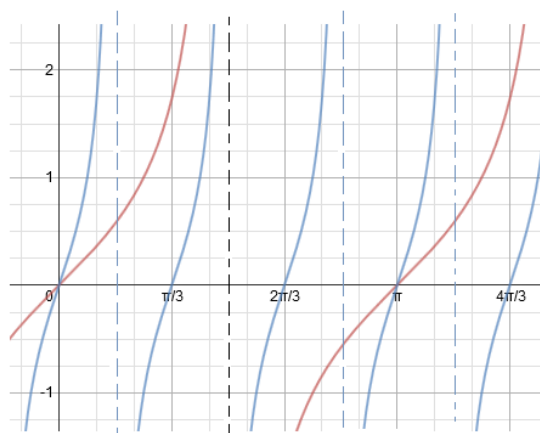
The curves and the asymptotes
shift 60° to the left

Note: The horizontal shift is the *opposite*
direction of the sign.

Period: Horizontal distance required for a periodic function
to complete one cycle.

Example 2: $y = \tan 3x$ Period: $\frac{\pi}{3}$

3 cycles between 0 and π



$x = \pi/6$ $x = \pi/2$ $x = 5\pi/6$ $x = 7\pi/6$ $\Leftarrow$ asymptotes

$$y = \tan Bx \longrightarrow \text{period} = \frac{\pi}{B}$$

For the parent function $y = \tan x$ (where $B = 1$) the period is π

As B increases, the period decreases.
In other words, it takes less time to complete one cycle.

And, for $y = \tan Bx$, as B decreases, the period increases.
In other words, it takes more time to complete one cycle.

NOTE: $\sin x$ and $\cos x$ periods are 2π , but $\tan x$ is π

$\tan x$ has one cycle... (period is π)

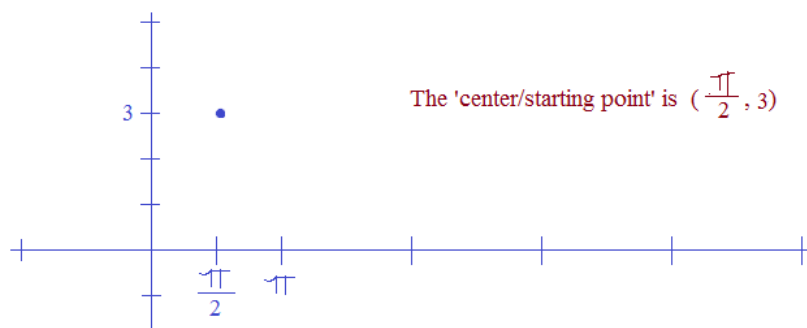
$\tan 3x$ has three cycles ... (period is $\pi/3$)

(so, 3 times as many asymptotes and intercepts)

Example: Sketch one period of the function $y = 2 \tan \frac{1}{4} (x - \frac{\pi}{2}) + 3$

The 'D' value is 3, so the vertical shift is up 3 units

The 'C' value is $\frac{\pi}{2}$, so the horizontal shift is $\frac{\pi}{2}$ to the right



$$y = A \tan B(x + C) + D$$

A: Amplitude (magnitude)

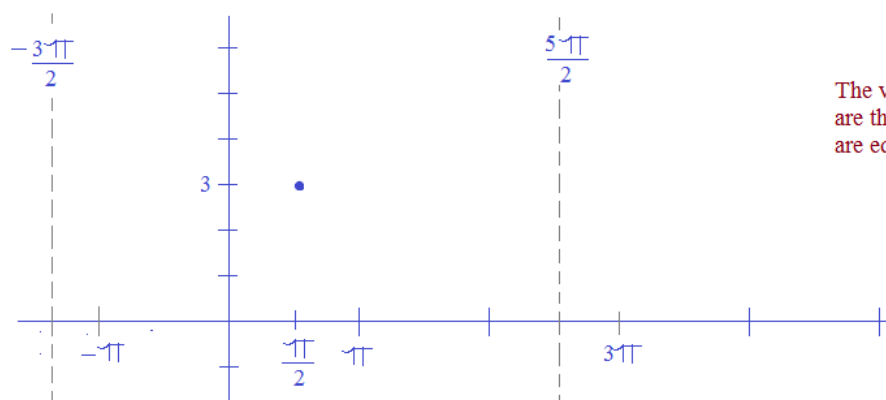
B: Period (cycles/ π)

C: Horizontal Shift

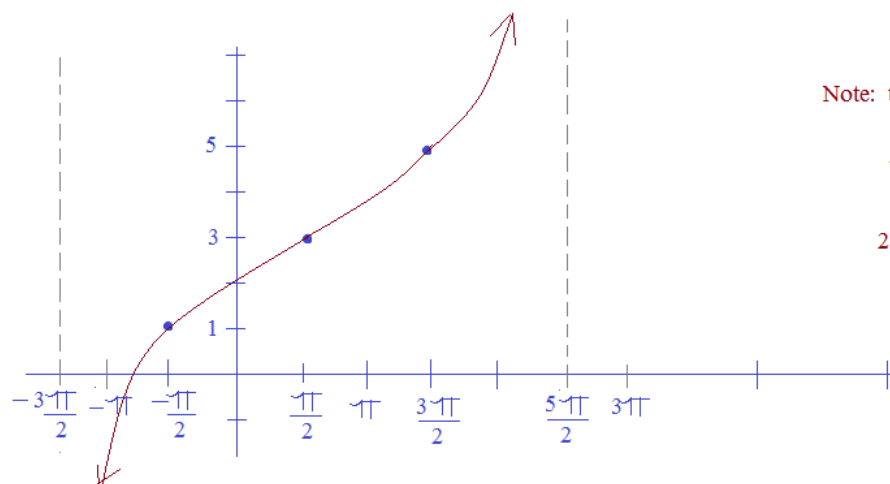
D: Vertical Shift

The 'B' value is $\frac{1}{4}$, so the period is $\frac{\pi}{\frac{1}{4}} = 4\pi$

The 'center point' is in the middle of the period... So, we can place vertical asymptotes 2π to the left... and, 2π to the right...

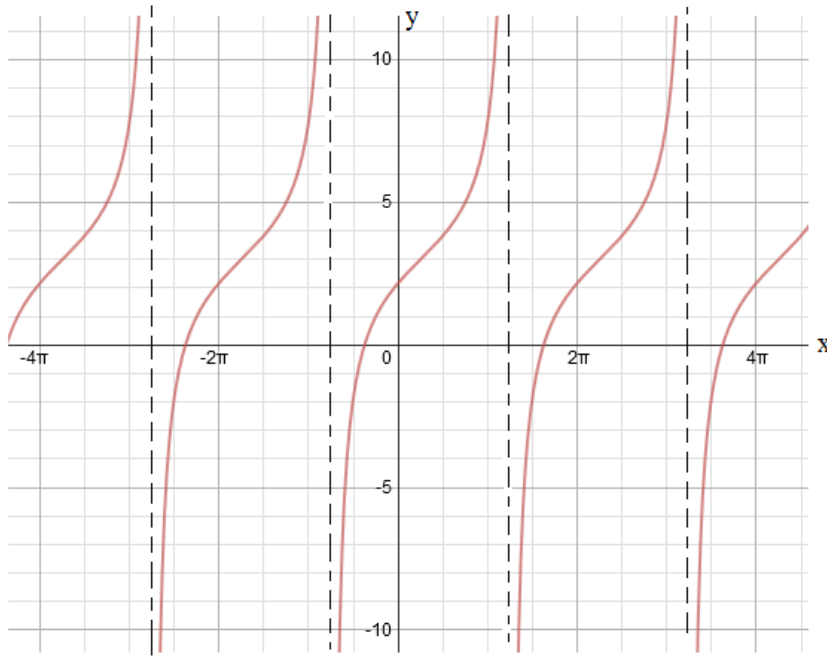


The 'A' Value is 2, so the "quarter values" will be up 2 and down 2...



Finally, use the 3 points and asymptotes to guide your sketch....

Example: Identify the following tangent function:



$$y = A \tan B(x + C) + D$$

A: Amplitude (magnitude)

B: Period (cycles/ π)

C: Horizontal Shift

D: Vertical Shift

Since the midpoint (i.e. the curve's point of inflection) is at 3, the vertical shift is UP 3

vertical shift (D): +3

One full cycle has a length of 2π . Since the period is 2π , $B = \frac{\pi}{2\pi} = \frac{1}{2}$

period (B): $\frac{1}{2}$

If the period ("B" value) is $1/2$, then the asymptotes would be at π , 3π , 5π etc..

But, in the graph, the asymptotes are at $\frac{5\pi}{4}$, $\frac{13\pi}{4}$, $\frac{21\pi}{4}$ $\longrightarrow$

horizontal shift to the right (C):
("phase") $-\frac{\pi}{4}$

In a $1/4$ cycle move, the value goes from 0 to 1... (i.e. $\tan(0) = 0$ $\tan(\frac{\pi}{4}) = 1$)

In the above graph, $1/4$ of a cycle is $\frac{\pi}{2}$... At $x = 0$, the output is 3

amplitude (A): 2

At $x = \frac{\pi}{2}$, the output is 5

This is an increase of 2 (instead of 1)

$$y = 2 \tan \frac{1}{2} \left(x - \frac{\pi}{4} \right) + 3$$

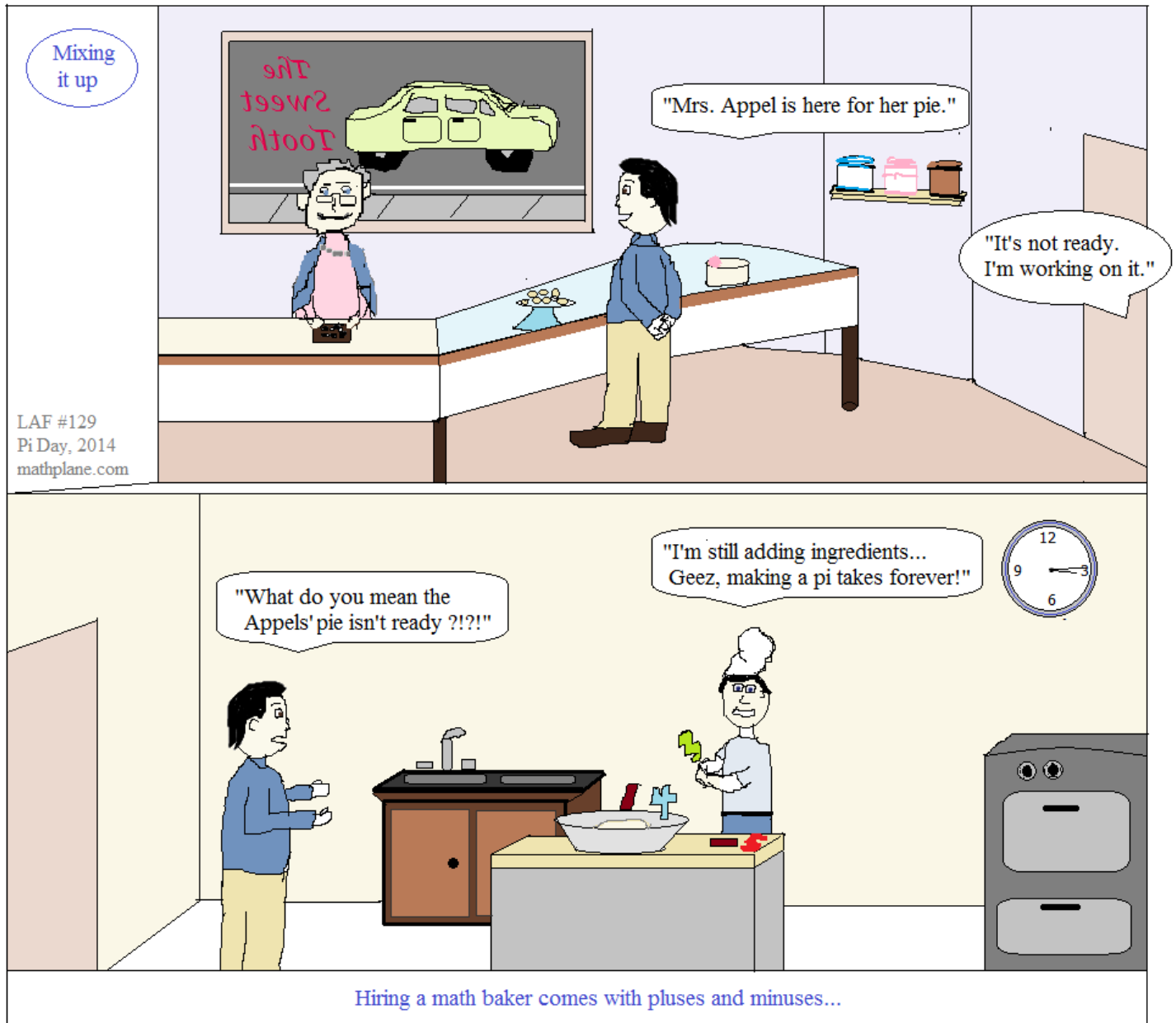
Test points to confirm your equation!

$$\text{If } x = \pi \quad 2 \tan \frac{1}{2} \left(\pi - \frac{\pi}{4} \right) + 3 = 2 \tan \frac{3\pi}{4} + 3$$

$$= 2 \cdot 2.41 + 3 \approx 7.8 \quad \checkmark$$

$$\text{If } x = -\frac{\pi}{2} \quad 2 \tan \frac{1}{2} \left(-\frac{\pi}{2} - \frac{\pi}{4} \right) + 3 = 2 \tan -\frac{3\pi}{4} + 3$$

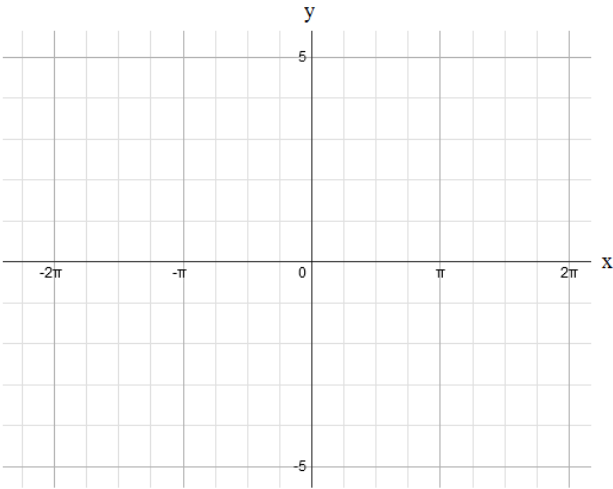
$$= 2 \cdot -2.41 + 3 \approx -1.8 \quad \checkmark$$



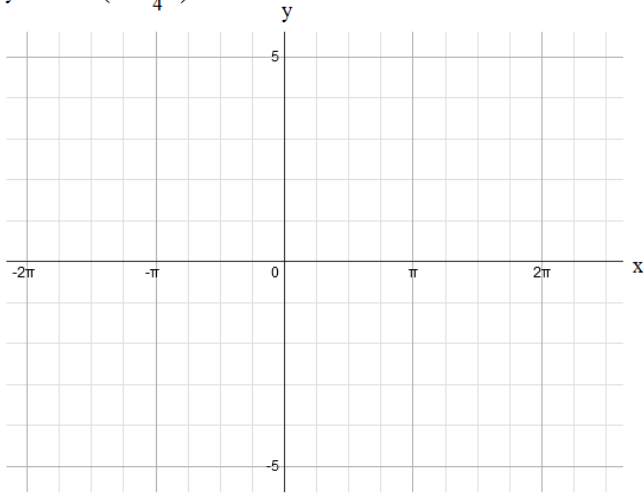
Practice ->

I. Graphing: Sketch each of the following equations. (Include at least 2 periods. And, label the asymptotes.)

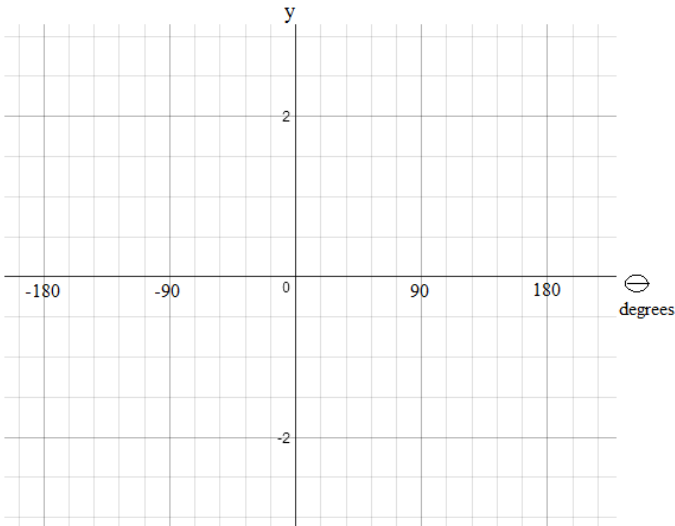
A) $y = 2\tan x - 3$



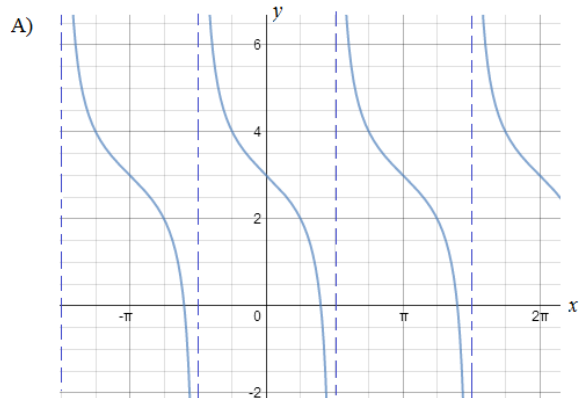
B) $y = 2 - \tan(x + \frac{\pi}{4})$



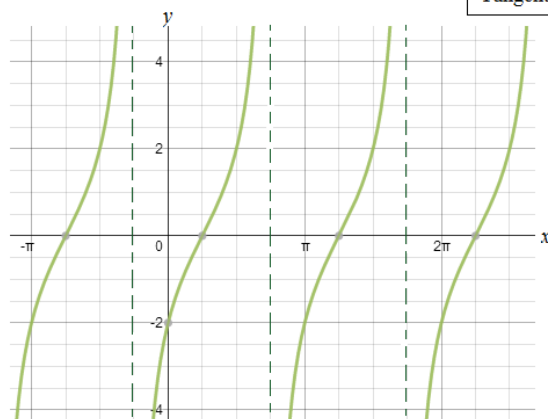
C) $y = \frac{\tan 2\theta}{3}$



II. Identifying: Determine the equations of the following.



B)



Tangent Function Practice

III. For the function $f(x) = \tan\left(x - \frac{\pi}{2}\right)$, determine the

- domain
- range
- maximum
- minimum
- x-intercepts (or, zeros)
- y-intercept

Challenge Question: Solve algebraically. Then, graph to confirm your solution.

$$\cos x = \tan x \quad (\text{in the interval } 0^\circ < x < 360^\circ)$$

"Remember the formula for success.
Check your work.
Go back over the tough questions.
Think of the reward for a job well done."

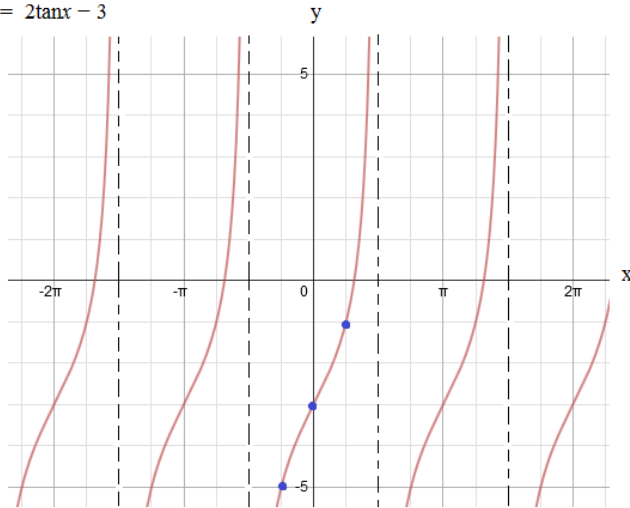


"Screw that..
The teacher gives partial credit.
Let's watch t.v."

Answers-→

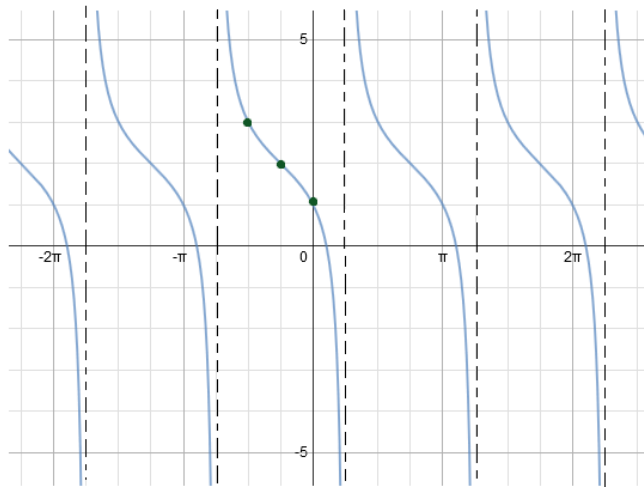
I. Graphing: Sketch each of the following equations. (Include at least 2 periods. And, label the asymptotes.)

A) $y = 2\tan x - 3$

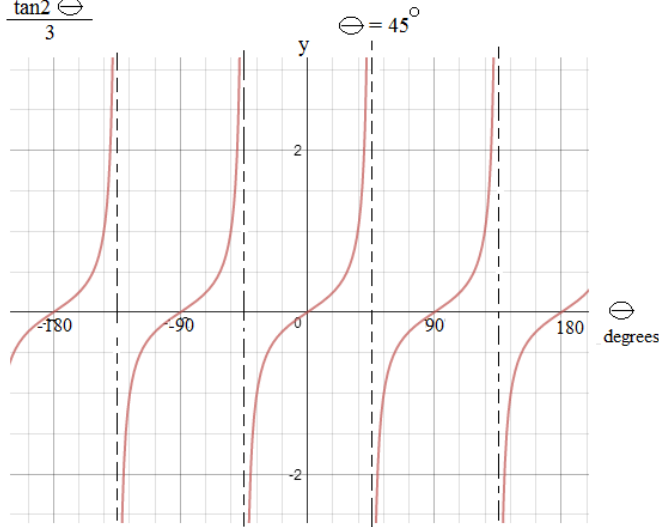


3 points: $(-\frac{\pi}{4}, -5)$ $(0, -3)$ $(\frac{\pi}{4}, -1)$

B) $y = 2 - \tan(x + \frac{\pi}{4})$



C) $y = \frac{\tan 2\Theta}{3}$



$$y = A \tan B(x + C) + D$$

A: Amplitude (magnitude)
B: Period (cycles/ π)
C: Horizontal Shift
D: Vertical Shift

Vertical shift (D): DOWN 3 units eg: a point of inflection at $(0, -3)$

Period (B = 1): π

Asymptotes at $x = -\frac{\pi}{2}$ $x = \frac{\pi}{2}$ $x = \frac{3\pi}{2}$ etc...

$$x = \frac{\pi}{2} + n\pi \text{ where } n \text{ is an integer}$$

Horizontal shift (C): None

Amplitude (A): Stretch by 2 at $\frac{\pi}{4}$ the function increases by 2 units (instead of 1)

rewrite the equation:

$$y = -\tan(x + \frac{\pi}{4}) + 2$$

"A" = -1 amplitude is 1, and the graph is reflected over the x-axis

"B" = 1 the period is $\frac{\pi}{1}$

"C" = $\frac{\pi}{4}$ the graph shifts $\frac{\pi}{4}$ to the left

"D" = 2 the graph shifts 2 units UP

3 points on the graph: $(-\frac{\pi}{2}, 3)$ $(-\frac{\pi}{4}, 2)$ $(0, 1)$

asymptotes: $x = \frac{\pi}{4} + n\pi$ where n is any integer

$$y = \frac{1}{3} \tan 2\Theta$$

"D" = 0 no vertical shift

"C" = 0 no horizontal shift

"A" = 1/3 the graph will "shrink" (up and down)

"B" = since B = 2, the period is cut in half.
It only takes 90 degrees for one cycle!

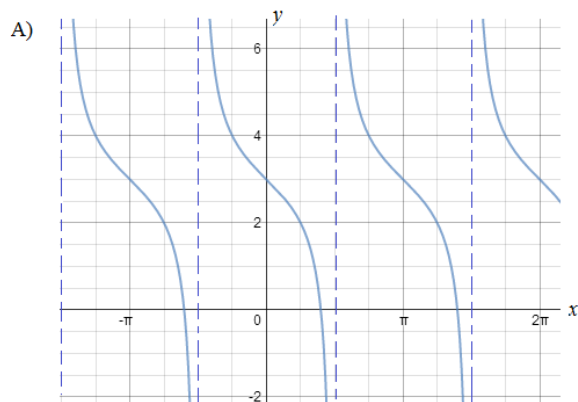
Since period is 90 degrees, there are asymptotes spaced 90 degrees apart...

when

$$\Theta = \dots -135^\circ, -45^\circ, 45^\circ, 135^\circ \dots$$

Test random points to check your answer!

II. Identifying: Determine the equations of the following.



Period is π $B = 1$

The 'center' or point of inflection occurs at $y = 3$ $y = -\tan x + 3$
at $y = 3$ Vertical shift $D = +3$

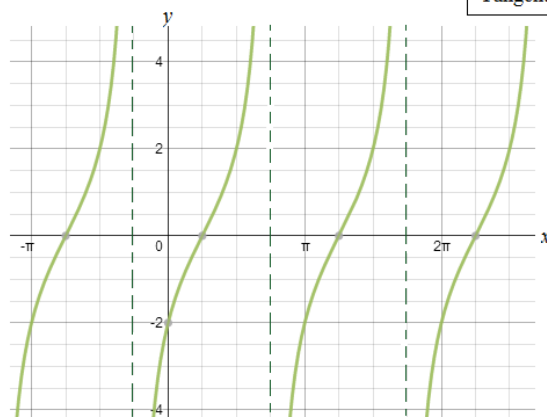
Asymptotes are at $\frac{\pi}{2} + \pi k$ No horizontal shift $C = 0$

At $\frac{\pi}{2}$, the graph moved down 1, so the amplitude $A = -1$

III. For the function $f(x) = \tan(x - \frac{\pi}{2})$, determine the

- domain all real numbers EXCEPT $\frac{\pi}{2} + \pi k$ (where k is an integer)
- range $(-\infty, \infty)$ i.e. all real numbers
- maximum None
- minimum None
- x-intercepts (or, zeros) $\frac{\pi}{2} + \pi k$ (where k is an integer)
- y-intercept Does not exist

B)



SOLUTIONS

Tangent Function Practice

$$y = 2\tan(x - \frac{\pi}{4})$$

Vertical asymptote moved from $-\frac{\pi}{2}$ to $-\frac{\pi}{4}$ horizontal shift right $C = -\frac{\pi}{4}$

Center/point of inflection is on x-axis No vertical shift $D = 0$

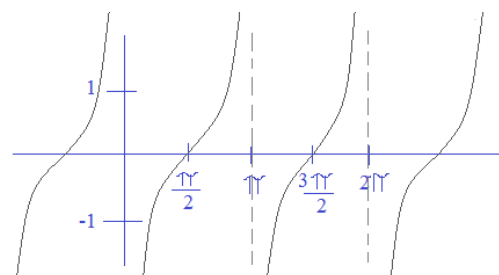
Period is π $B = 1$

at $\frac{\pi}{4}$ output is 0 at $\frac{\pi}{2}$ output is 2 (instead of 1)
this implies the amplitude is $A = 2$

Using a sketch and table of values as a guide:

x	y
0	undefined
$\frac{\pi}{4}$	-1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined
$\frac{5\pi}{4}$	0

(tangent function shifted $\frac{\pi}{2}$ to the right)



Challenge Question: Solve algebraically. Then, graph to confirm your solution.

$$\cos x = \tan x \quad (\text{in the interval } 0^\circ < x < 360^\circ)$$

$$\frac{\cos x}{1} = \frac{\sin x}{\cos x} \quad (\text{Trig identity})$$

$$\cos^2 x = \sin x \quad (\text{Cross multiply})$$

$$1 - \sin^2 x = \sin x \quad (\text{Trig identity})$$

$$\sin^2 x + \sin x - 1 = 0 \quad (\text{Quadratic formula})$$

$$\sin x = .618 \text{ and } -1.618$$

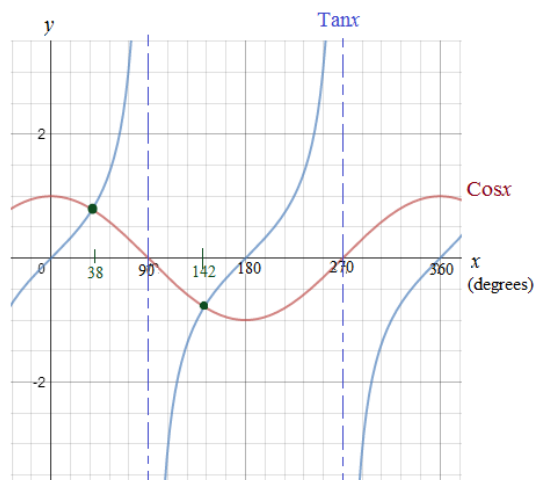
$$x = 38.2 \text{ degrees} \quad (-1.618 \text{ is extraneous...})$$

Then, since tangent and cosine have same signs ('negative') in Quadrant II

$$\text{Reference angle: } 38.2 \text{ degrees} \rightarrow 180 - 38.2 = 141.8 \text{ degrees}$$

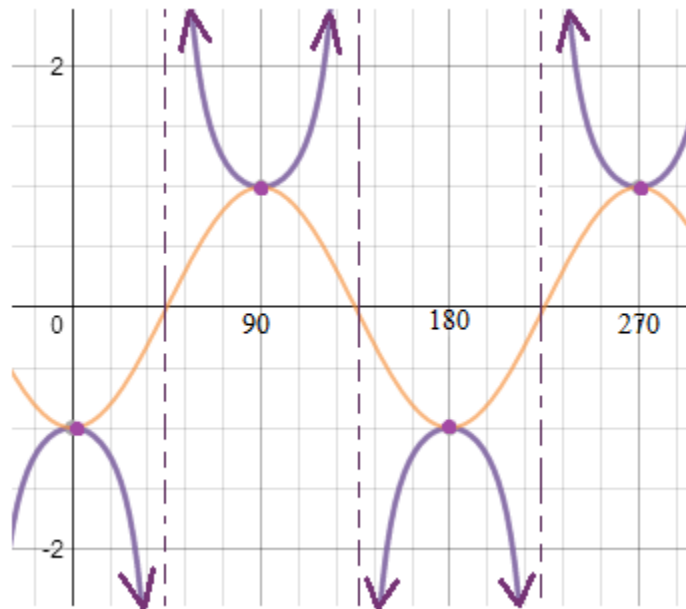
$$\text{Quick check: } \cos(38.2) = .786 \text{ (approx)}$$

$$\tan(38.2) = .787 \text{ (approx)}$$



The two points of intersection (in the interval) represent the solution to the equation.

Reciprocal Functions



Topics include sketching secant, cotangent, & cosecant, solving systems, transformations, and more.

Periodic trig functions: "The reciprocals"

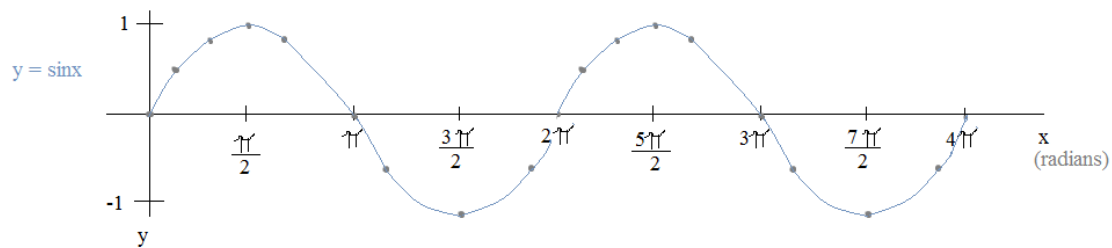
Review:

$$y = \sin \ominus$$

The following is a table of chosen values:

$\ominus$	0	30	60	90	120	180	210	270	330	360
y	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0

Then, plot the points on a graph....



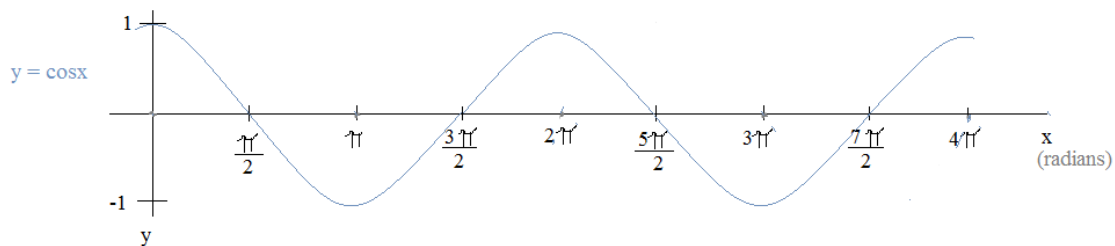
A cycle is a complete pattern repetition. This graph contains 2 cycles.

The period is 2π (Horizontal length of one cycle)

$$y = \cos \ominus$$

The following is a table of chosen values:

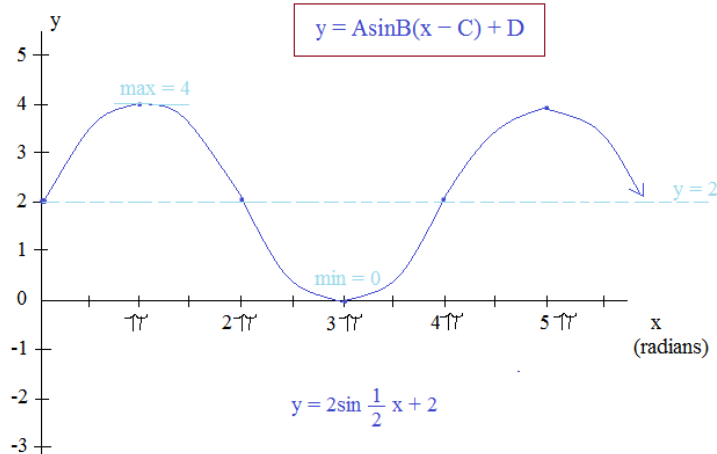
$\ominus$	0	30	60	90	120	180	210	270	330	360
y	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$	1



A cycle is a complete pattern repetition. This graph contains 2 cycles.

The period is 2π (Horizontal length of one cycle)

Example I:



D (vertical shift) : The center that the sine wave is oscillating over is $y = 2$. Therefore, the vertical shift is up 2 units.

A (amplitude) : The maximum y-value is 4, and the minimum y-value is 0 --- a total span of 4 units. The amplitude is 1/2 of that amount: 2 units

C (horizontal shift) : Since $y = 2$ at 0 radians, there is no horizontal shift.

B (period) : The horizontal distance of one cycle is 4π . Since $\frac{2\pi}{B} = 4\pi$, $B = \frac{1}{2}$

Example II:

$$y = A \cos B(x - C) + D$$

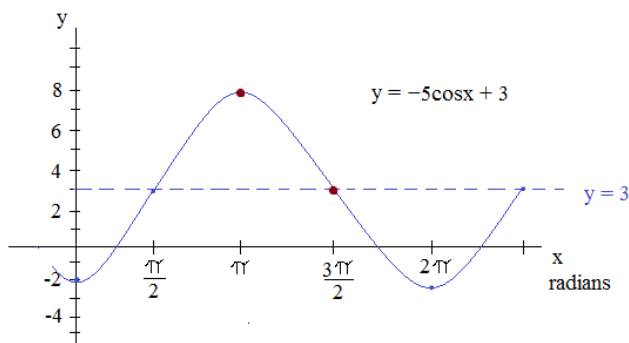
amplitude: $A = -5$
(negative, so "faces down")

period: 2π

Vertical shift: $D = 3$
Up 3

Horizontal shift: C
none..

max 8 ; min -2



Check: $x = \pi$

$$x = \frac{3\pi}{2}$$

At $x = \pi$,

$$y = -5\cos(\pi) + 3 = -5(-1) + 3 = 8 \quad \checkmark$$

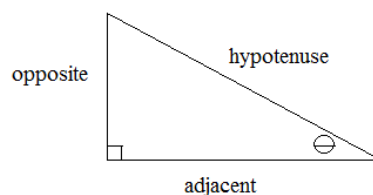
At $x = \frac{3\pi}{2}$

$$y = -5\cos\left(\frac{3\pi}{2}\right) + 3 = -5(0) + 3 = 3 \quad \checkmark$$

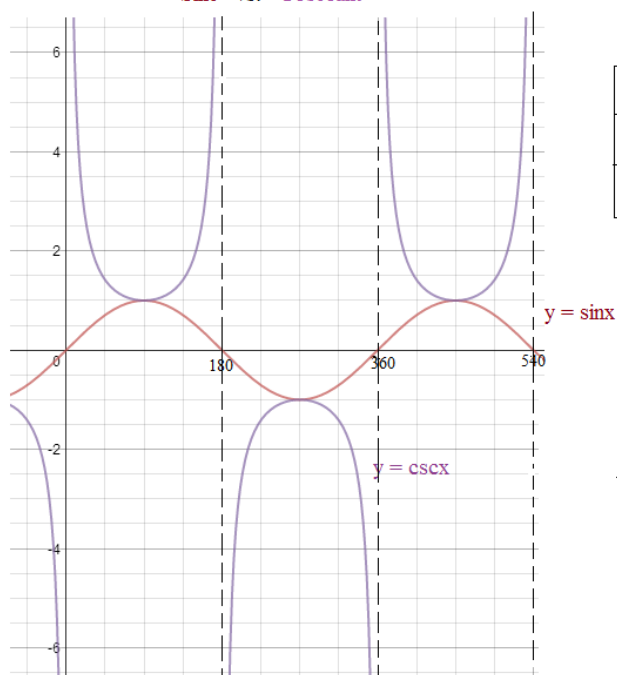
$$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}} \Rightarrow \text{cosecant} = \frac{1}{\text{sine}} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}} \Rightarrow \text{secant} = \frac{1}{\text{cosine}} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}} \Rightarrow \text{cotangent} = \frac{1}{\text{tangent}} = \frac{\text{adjacent}}{\text{opposite}}$$



Sine vs. Cosecant



$\ominus$ degrees	0	30	60	90	100	120	150	170	175	180
Sine	$\frac{0}{1}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	.985	.866	.5	.174	.087	0
Cosecant	undefined	$\frac{2}{1}$	$\frac{2}{\sqrt{3}}$	1	1.015	1.155	2	5.759	11.47	undefined

$$\sin \ominus = \frac{1}{\csc \ominus}$$

$\ominus$ degrees	180	210	240	270	300	315	330	340	350	360
Sine	$\frac{0}{1}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	-.866	-.707	-.5	-.342	-.174	0
Cosecant	undefined	$-\frac{2}{1}$	$-\frac{2}{\sqrt{3}}$	-1	-1.155	-1.414	-2	-2.92	-5.759	undefined

NOTE: Where there is a zero for $\sin x$, there is an asymptote for $\csc x$

Observation: The intersections of $\sin x$ and $\csc x$ are at $y = 1$ and $y = -1$

Sketching a reciprocal trig function

Example: Sketch $y = \sec x$ on the interval $[0, 4\pi]$

Step 1: Sketch the 'original' function

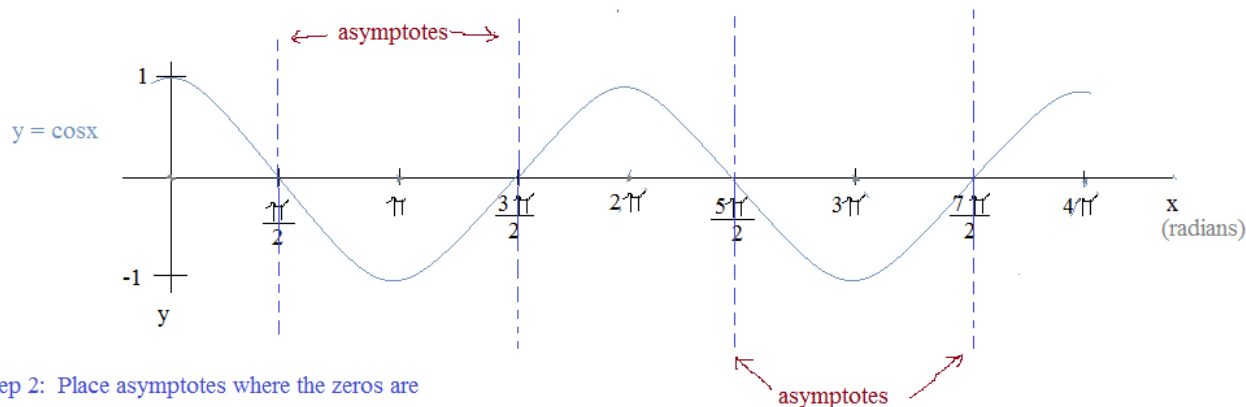
The reciprocal of $\sec x$ is $\cos x$:

Sketch the 'original' trig function.

Place asymptotes where the zeros are.

Identify the maximums/minimums

Plot easy points and extend.



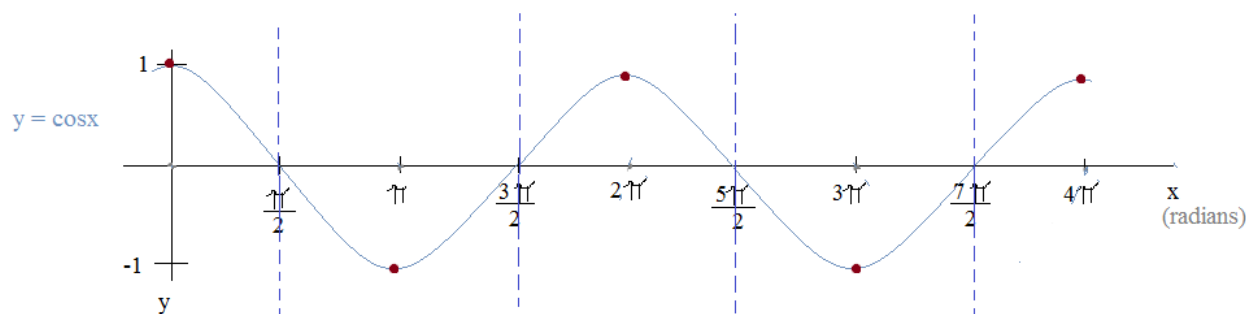
Step 2: Place asymptotes where the zeros are

Step 3: Maximums and Minimums are the same

$$\cos(0) = \sec(0) = 1$$

$$\cos(\pi) = \sec(\pi) = -1$$

$$\cos(2\pi) = \sec(2\pi) = 1 \quad \text{etc...}$$

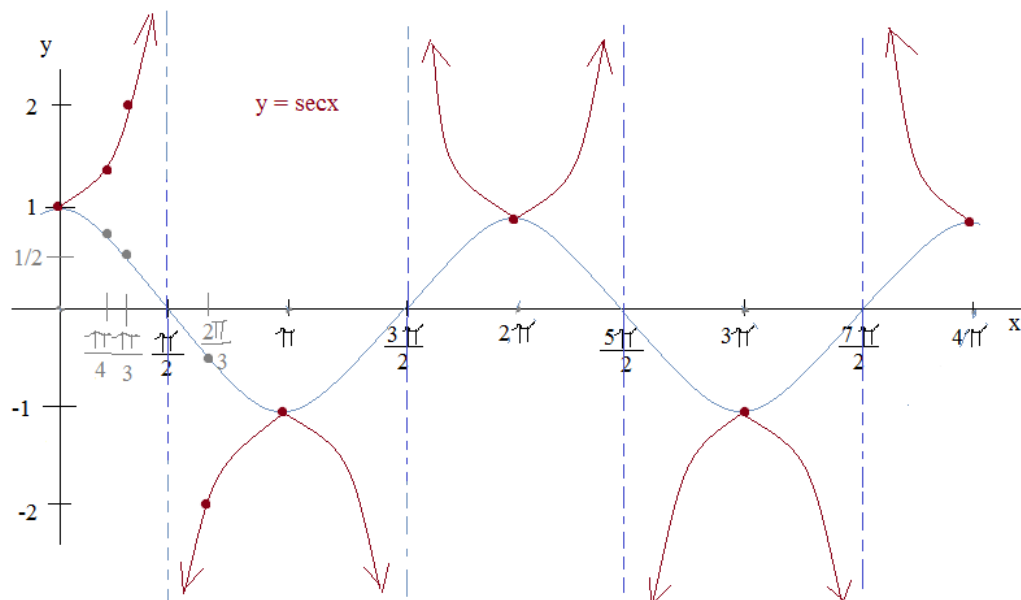


Step 4: Plot easy points and extend

We know (from the unit circle) that $\cos(\pi/3) = 1/2$ And, so $\sec(\pi/3) = 2/1$

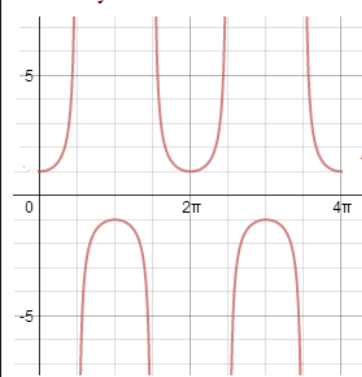
$\cos(2\pi/3) = -1/2$ And, so $\sec(2\pi/3) = -2/1$

$\cos(\pi/4) = .707$ (approx.) then, $\sec(\pi/4) = 1.41$ (approx)



Graphing Calculator:

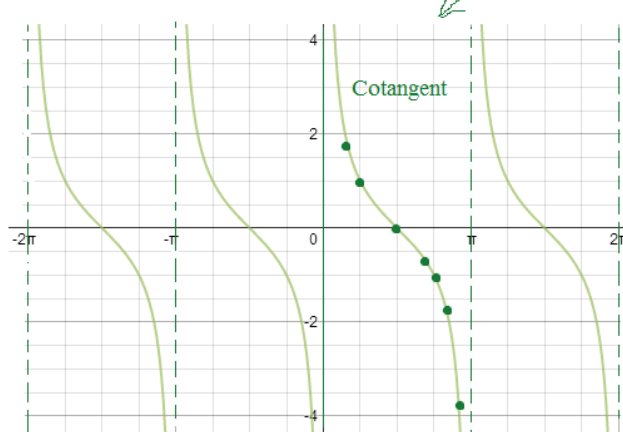
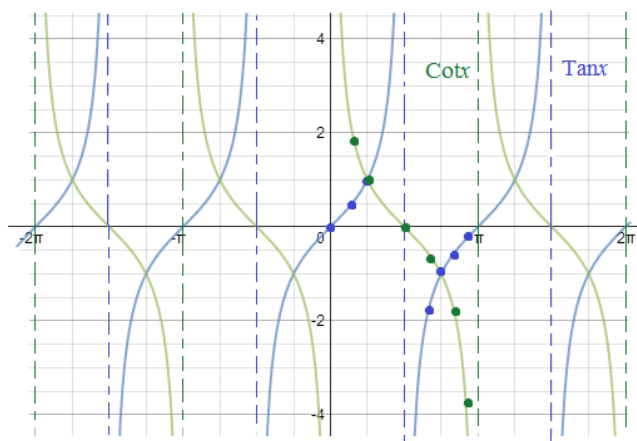
$y = \sec x$



Sketching Cotangent Functions

Using a sample of points, we can discover a pattern...

x radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
Tanx	0	$\frac{1}{\sqrt{3}}$	1	undefined	-1.732	-1	-0.577	-0.268	0
Cotx	undefined	$\frac{\sqrt{3}}{1}$	1	0	-0.577	-1	-1.732	-3.732	undefined



Observation: If you take the graph of **Tanx**, reflect it over the x-axis and shift $\frac{\pi}{2}$ to the left (or right), it becomes **Cotx**

$$\text{Cot}x = -\text{Tan}(x + \frac{\pi}{2})$$

Example: Sketch the function $f(x) = \cot(\frac{1}{4}x) + 2$

Recognizing the general periodic equation: $y = A \cot(B(x - C)) + D$

A: Amplitude is 1 (the shape of the function remains)

B: $\frac{1}{4}$ so, it takes 4π to move one period

$$\text{period} = \frac{\pi}{B}$$

C: Horizontal shift is 0

D: Vertical shift is UP 2 units

$\text{Cot}(0) = \frac{1}{0}$ and, since there is no horizontal shift, asymptotes are placed at the beginning (or end) of each period...

asymptotes at $\dots -4\pi, 0, 4\pi \dots$

The 'center' of each period is the midpoint between the asymptotes and, because there is a vertical shift of +2

$y = 2$ at $\dots, -2\pi, 2\pi, \dots$

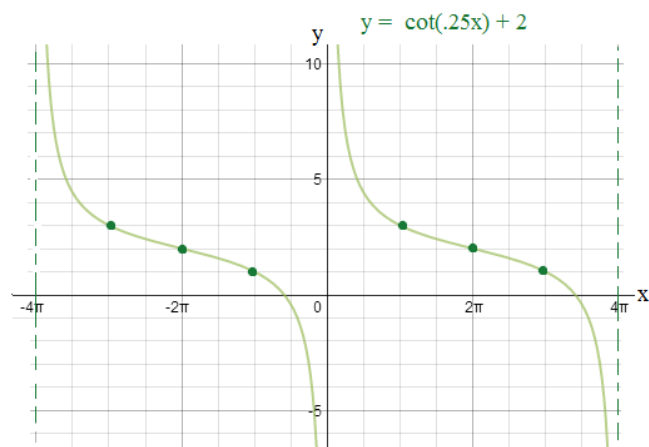
Then, since the amplitude is 1, the 'quarter' points will be up 1 unit and down 1 unit....

$y = 3$ at $\dots, -3\pi, \pi, \dots$

$y = 1$ at $\dots, -\pi, 3\pi, \dots$

Sketching Cotangent and Tangent Functions

- 1) Identify the parts
- 2) Draw the vertical asymptotes
- 3) Plot the points of inflection (midpoints of each period)
- 4) Plot the 'quarter points'
- 5) Extend the curves



Graphing a cosecant function with transformations

Example: Sketch $y = 3\csc\frac{1}{2}(x - 60^\circ) + 4$

Step 1: Lightly sketch the 'original' function (i.e. reciprocal of this reciprocal)

$$y = 3\sin\frac{1}{2}(x - 60^\circ) + 4$$

Amplitude: 3

Period: $\frac{2(180)}{\frac{1}{2}} = 720$ degrees

Horizontal shift: 60 degrees to the right ("phase")

Vertical shift: UP 4 units

Sketch the 'original' trig function.
Place asymptotes where the zeros are.
Identify the maximums/minimums
Plot easy points and extend.

Step 2: Place asymptotes where the zeros are...

Since the equation is shifted up 4 units,
the "zeros" are at $y = 4$

Step 3: Identify the maximums and minimums

Since the amplitude is 3, the maximums are 3 above the 'wave center' and the minimums are 3 below the center.

maximums: where $y = 7$

minimums: where $y = 1$

Step 4: Plot a few points and extend

$$y = 3\csc\frac{1}{2}(x - 60^\circ) + 4$$

If $x = 360$ degrees

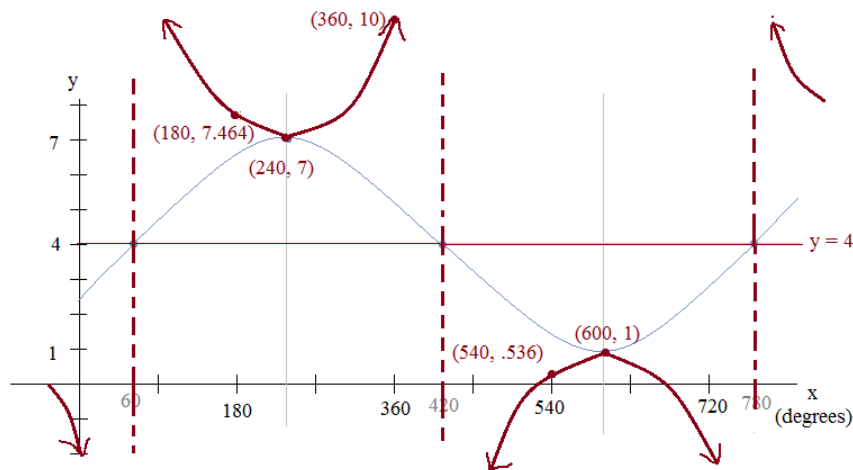
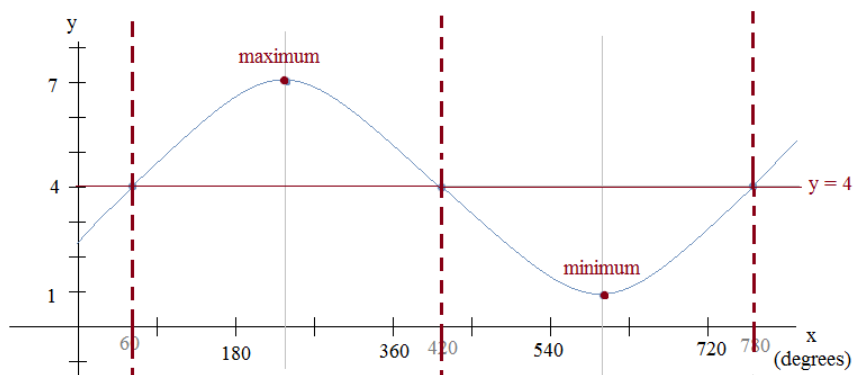
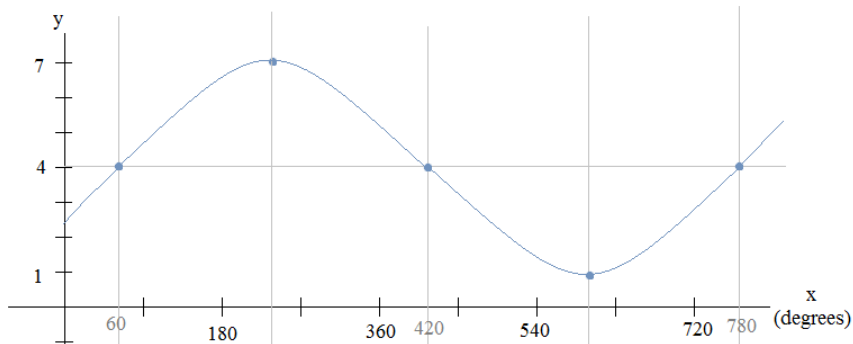
$$= 3\csc(150^\circ) + 4 = 3(2) + 4 = 10$$

If $x = 540$ degrees

$$= 3\csc(240^\circ) + 4 = 3\left(-\frac{2}{\sqrt{3}}\right) + 4 = .536$$

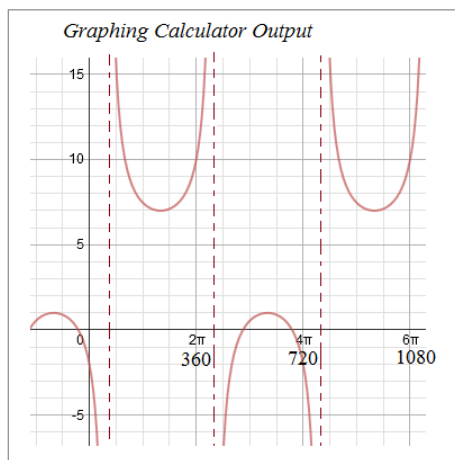
If $x = 180$ degrees

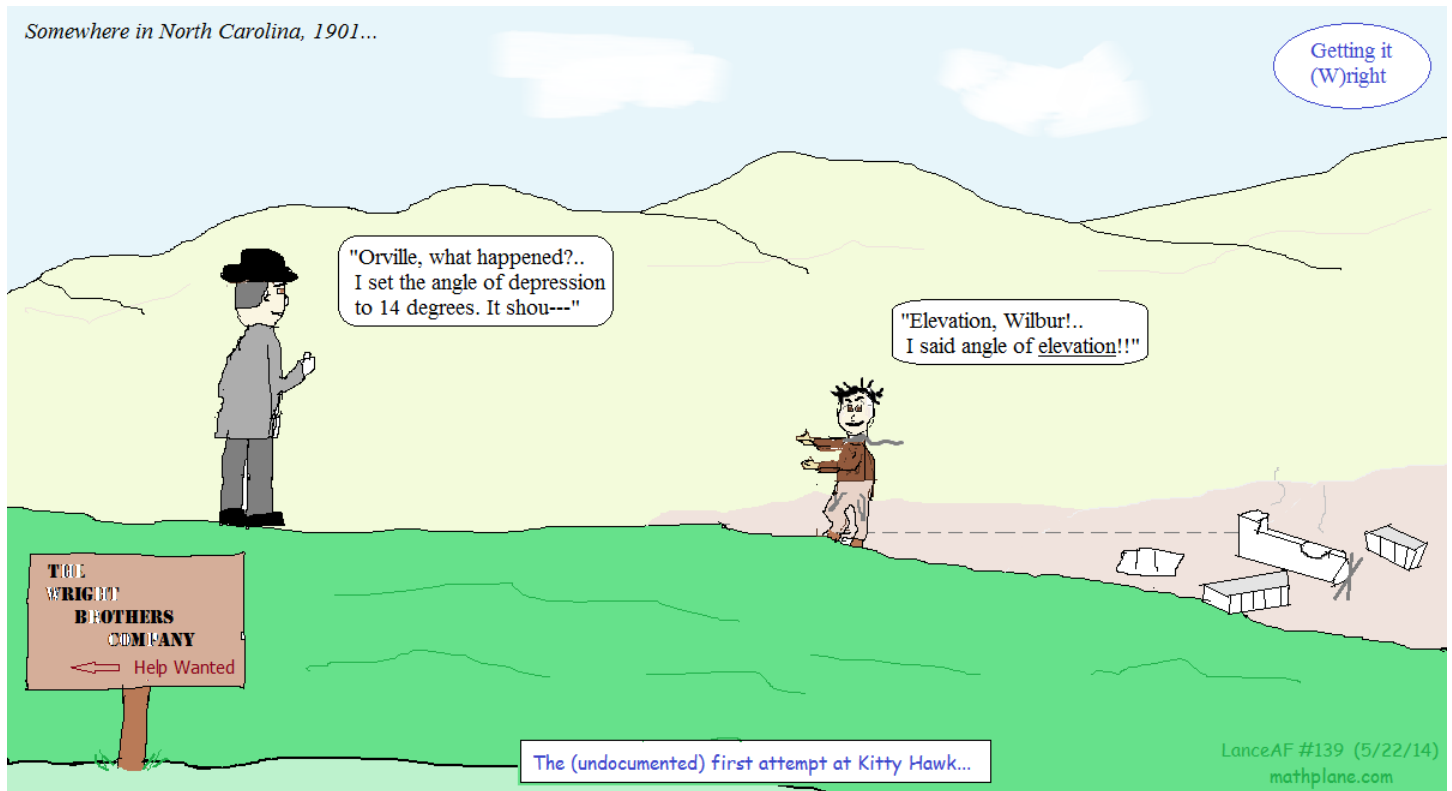
$$= 3\csc(60^\circ) + 4 = 3\left(\frac{2}{\sqrt{3}}\right) + 4 = 7.464$$



Note: the maximum and minimum points of sine are also points in cosecant.

(240, 7) and (600, 1)



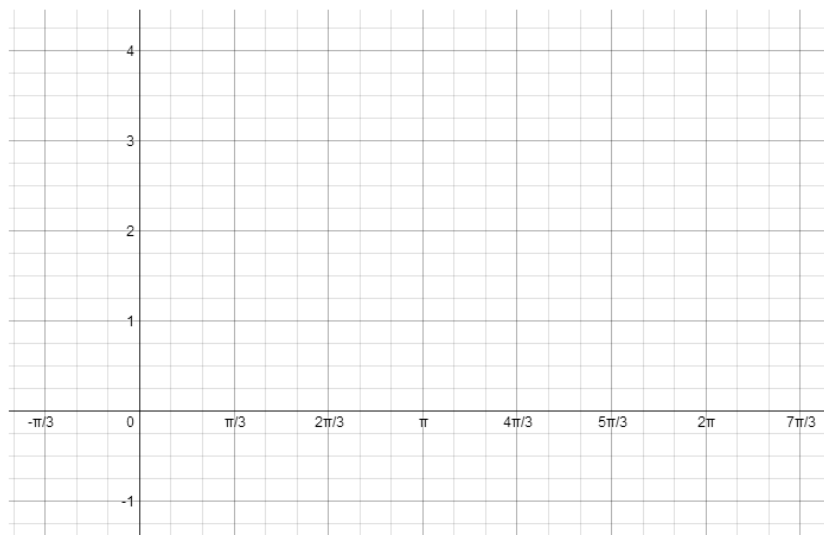


Practice ->

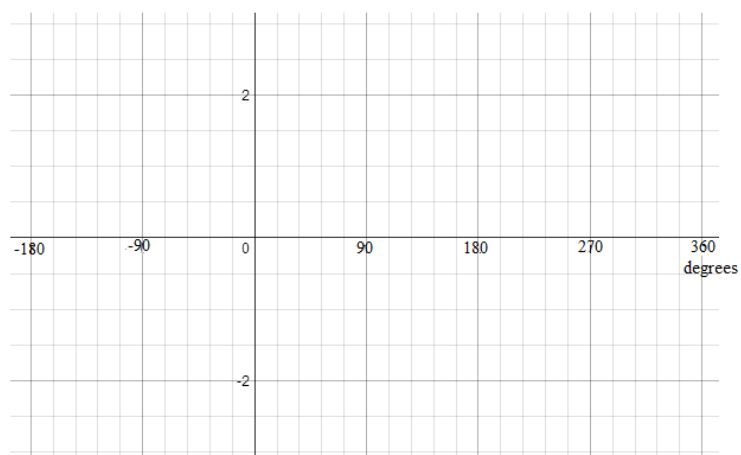
Quick Quiz: (Periodic) Trig Reciprocal Functions

I. Graphing: Sketch at least 1 period for each function. Label asymptotes and points.

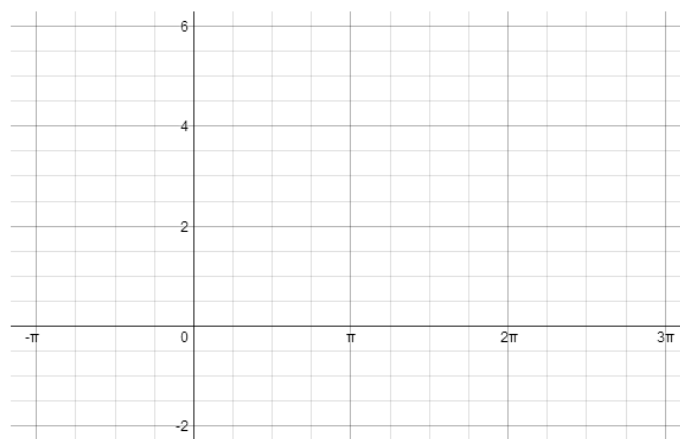
A) $y = \csc(x - \frac{\pi}{3}) + 2$



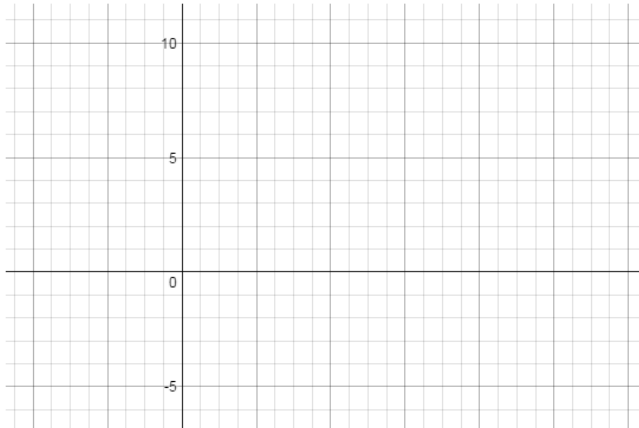
B) $y = -\sec 2\theta$



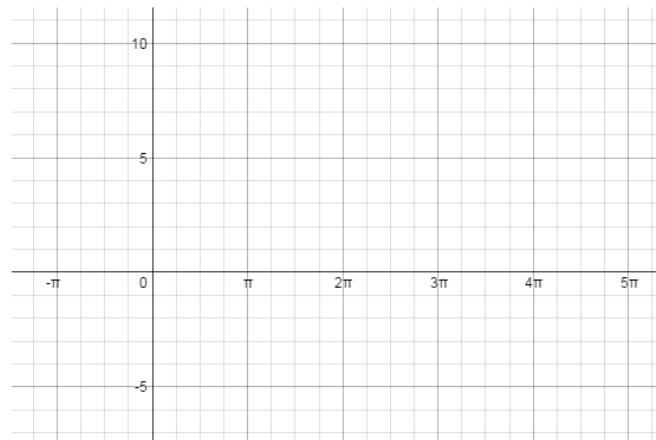
C) $y = \frac{1}{2} \cot(x + \frac{\pi}{3}) + 3$



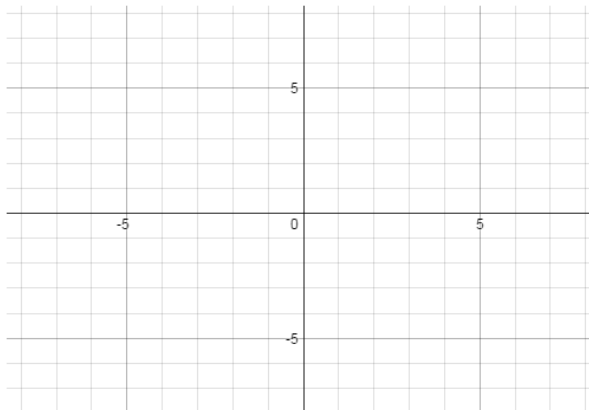
D) $3\csc 2(x - \frac{\pi}{4}) + 4$



E) $-2\sec \frac{1}{2}(x + \frac{\pi}{4}) + 5$

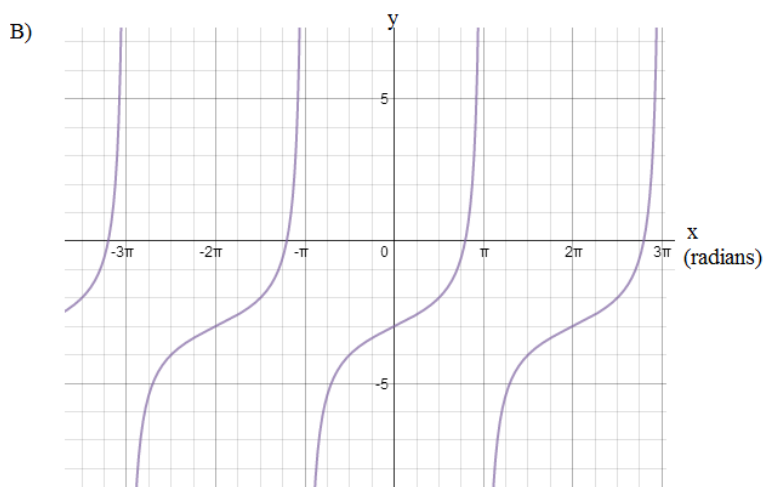
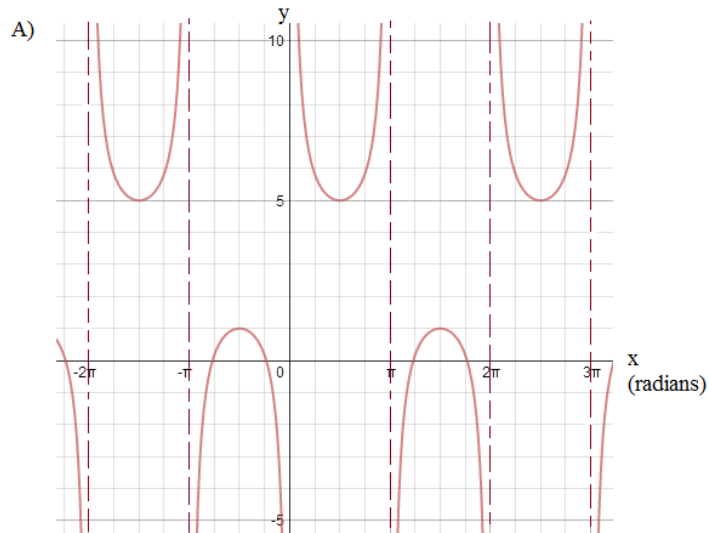


F) $3\tan \frac{\pi}{4}(x - 2)$



Quick Quiz: (Periodic) Trig Reciprocal Functions

II. Identifying Graphs: Write an equation that describes the sketch. (**Extra Credit: Write a 2nd equation, using a different trig function.)

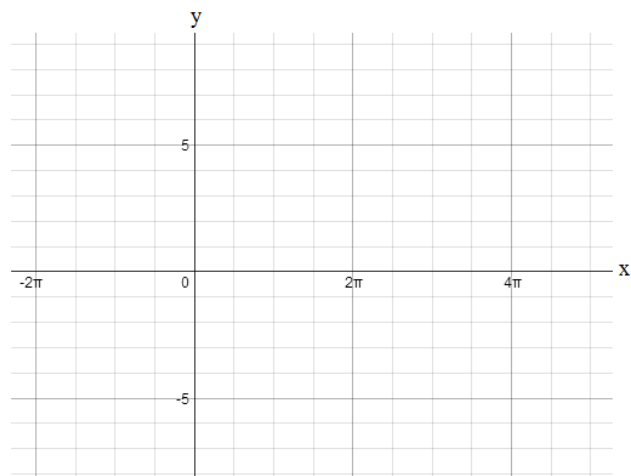


III. System of equations

Solve the following algebraically. Then, graph to confirm your solutions.

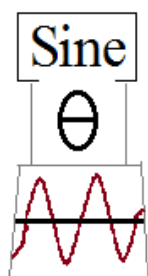
$$y = 2\sin x + 3$$

$$y = 5\csc x$$



BEST
FRIEND

"I know I can always count on you..
.. even during my negative times..."



"Absolutely"

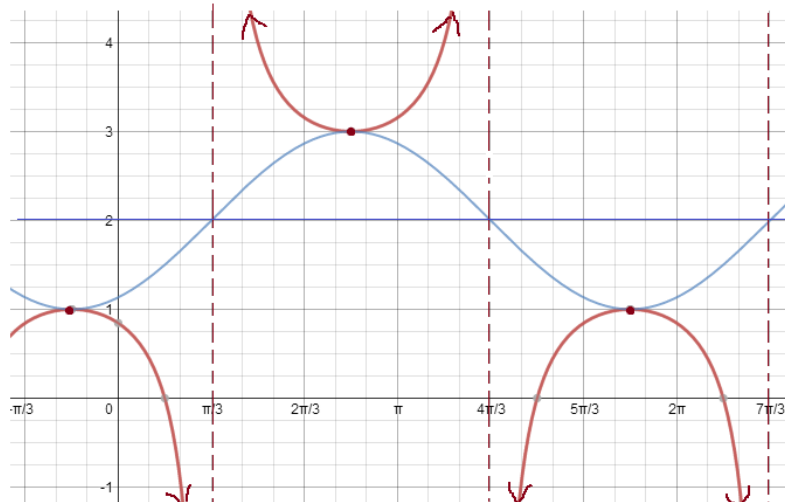
|HE|

A true optimist: HE always stays positive....

Answers-→

I. Graphing: Sketch at least 1 period for each function. Label asymptotes and points.

A) $y = \csc(x - \frac{\pi}{3}) + 2$ (the red curves)



Step 1: Sketch the sine function

$$y = \sin(x - \frac{\pi}{3}) + 2$$

Vertical shift: UP 2 Period: 2π

Horizontal shift: $\frac{\pi}{3}$ to the right Amplitude: 1

Step 2: Set the asymptotes at the zeros (where the points of inflections are)

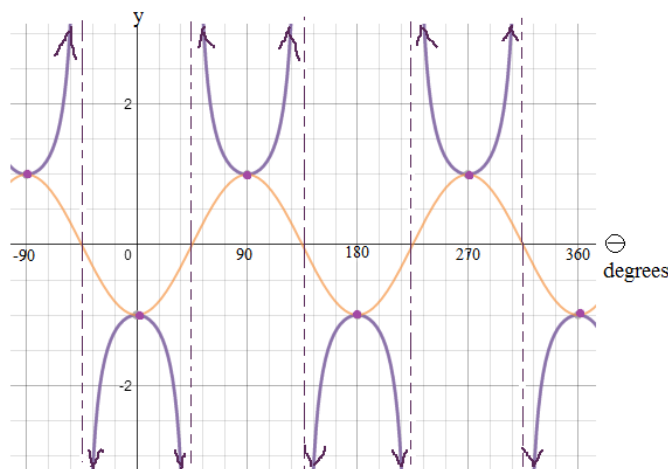
$$\text{asymptotes at } x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \dots$$

Step 3: Recognize the (sine) maximum and minimums

$$(-\frac{\pi}{6}, 1) \quad (\frac{5\pi}{6}, 3) \quad (\frac{11\pi}{6}, 1) \text{ etc...}$$

Step 4: Extend the curves

B) $y = -\sec 2\theta$ (The purple curves)



Step 1: Sketch $y = -\cos 2\theta$

No vertical shift; no horizontal shift...

period is 180 degrees; amplitude is 1 AND since it is negative, reflected over the x-axis

Step 2: Set asymptotes

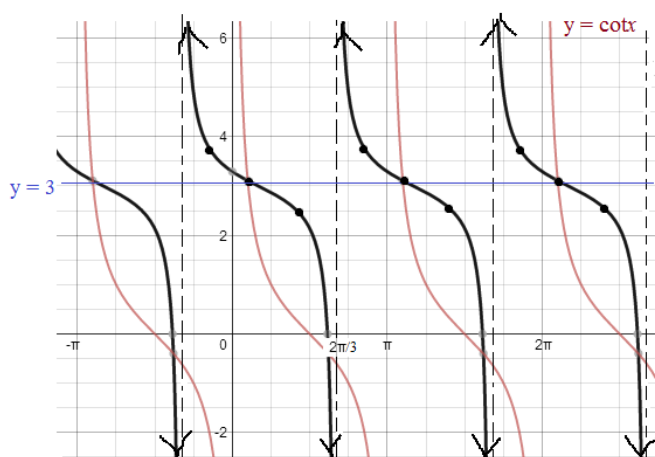
Asymptotes occur where $\cos 2\theta = 0$

$$@ \dots, -45^\circ, 45^\circ, 135^\circ, 225^\circ, 315^\circ, \dots$$

Step 3: Recognize the maximums and minimums (where $y = 1$ or $y = -1$)

Step 4: Extend the curves (and, check points)

C) $y = \frac{1}{2} \cot(x + \frac{\pi}{3}) + 3$ (black curves)



Recognizing the general periodic equation: $y = A \cot B(x - C) + D$

Amplitude is 1/2 Period is π Vertical shift: UP 3 units

Horizontal shift: $\frac{\pi}{3}$ to the left Asymptotes shift as well...

Then, sketch the transformation:

Asymptotes move to $-\frac{\pi}{3} + \pi k$

Plot the key points:

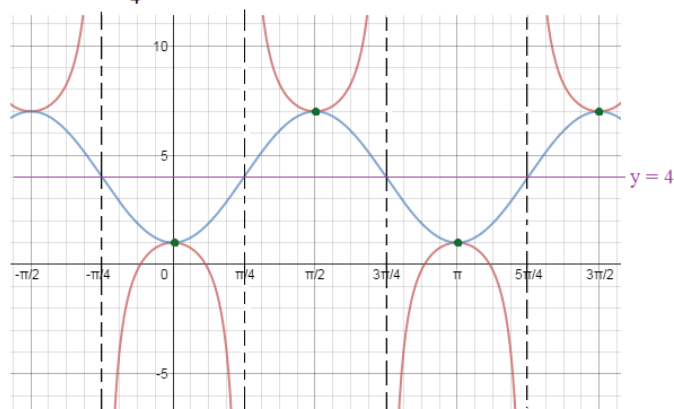
The 'center' moves from $(\frac{\pi}{2}, 0)$ to $(\frac{\pi}{6}, 3)$

From the center, $\frac{\pi}{4}$ to the right ----> decrease by 1/2 (amplitude is 1/2)
 $\frac{\pi}{4}$ to the left ----> increase by 1/2

Fill in points in each period... Extend the curves...

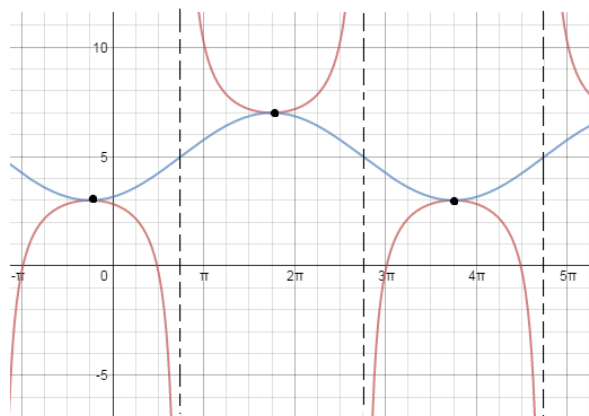
D) $3\csc 2(x - \frac{\pi}{4}) + 4$

SOLUTIONS



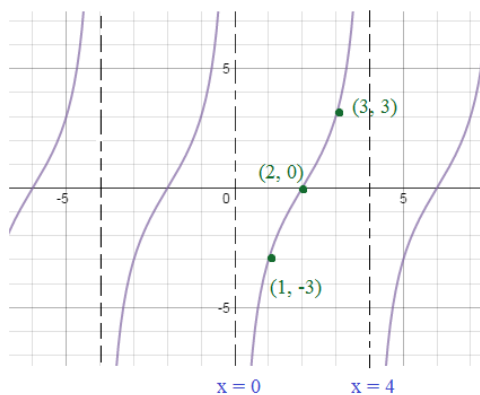
one period (π) is cut into "quarters"

E) $-2\sec \frac{1}{2}(x + \frac{\pi}{4}) + 5$



one period (4π) spans from $-\frac{\pi}{4}$ to $\frac{15\pi}{4}$

F) $3\tan \frac{\pi}{4}(x - 2)$



Quick Quiz: (Periodic) Trig Functions

Step 1: sketch sine equivalent

$$3\csc 2(x - \frac{\pi}{4}) + 4$$

A B C D

A: Vertical stretch: 3

B: Period: $\frac{2\pi}{B} = \pi$

C: Horizontal ("Phase") Shift:

$\frac{\pi}{4}$ to the right

D: Vertical Shift: up 4 units

Step 2: place asymptotes at the "zeros"

vertical asymptotes will be placed where $y = 4$

Step 3: recognize the extremes of the sine functions

$(0, 1)$ $(\frac{\pi}{2}, 7)$ $(\pi, 1)$ $(\frac{3\pi}{2}, 7)$

Step 4: extend the curves to finish the sketch

Sketch the equivalent cosine function

$$-2\cos \frac{1}{2}(x + \frac{\pi}{4}) + 5$$

A B C D

A: Vertical stretch: 2
(since it is negative, it reflects over the x-axis)

B: Period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$

C: Horizontal shift $\frac{\pi}{4}$ to the left

D: Vertical shift: up 5 units

Period starts at $-\frac{\pi}{4}$ then, it ends at $-\frac{\pi}{4} + 4\pi$

the midpoint is $\frac{7\pi}{4}$ and the "quarter points" are

then, place asymptotes at zeros
and recognize the max/min...
finally, extend the curves...

$\frac{3\pi}{4}$ $\frac{11\pi}{4}$

Step 1: Identify the parts

A: vertical stretch is 3

C: horizontal shift:
2 units to the right

B: period is $\frac{\pi}{\frac{\pi}{4}} = 4$

D: vertical shift: 0

Step 2: Place the center point

since the vertical shift is 0 and the horizontal shift is +2,
the "center point" is (2, 0)

Step 3: Place asymptotes ("1/2 to the left and 1/2 to the right")

Since the period is 4, we'll place an asymptote 2 units
to the left of the "center" ---> $x = 0$

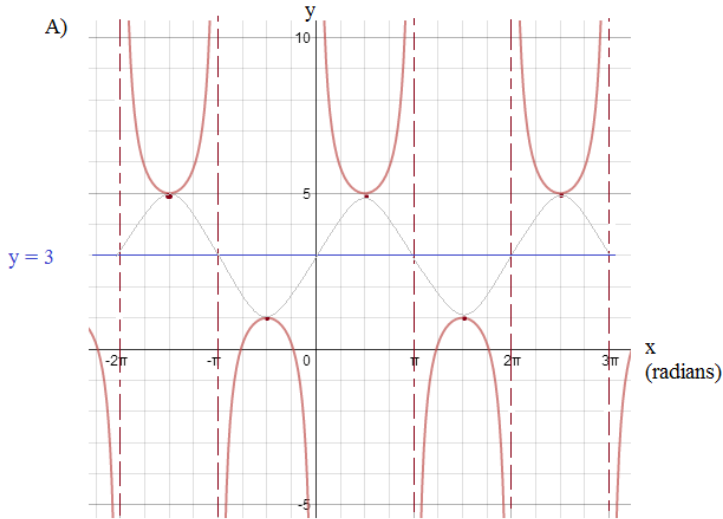
and, we'll place an asymptote 2 units to the
right of the "center" ---> $x = 4$

Step 4: Place "quarter points"

Since the period extends from 0 to 4, the quarter points
are at $x = 1$ and $x = 3$...

also, since the vert. stretch is 3, the "quarter points" will
be up 3 and down 3... (3, 3) and (1, -3)

II. Identifying Graphs: Write an equation that describes the sketch. (**Extra Credit: Write a 2nd equation, using a different trig function.)



Since the curves are split (the range isn't all real numbers), it appears to be cosecant or secant...

Period: Each cycle is 2π $B = 1$

Vertical shift: The wave 'center' is between $y = 5$ and $y = 1$

Center is at $y = 3$ $D = +3$

Amplitude: The distance between center ($y = 3$) and the upper curves ($y = 5$) and lower curves ($y = 1$) is 2 units..

$A = 2$

Horizontal shift: Depends on the function...

There is an asymptote at $x = 0$

Therefore, the reciprocal would have a zero at $x = 0$

$$\sin(0) = 0 \dots \quad y = 2\csc(x) + 3$$

$$y = 2\sec\left(x - \frac{\pi}{2}\right) + 3$$

Each period is continuous, so it appears to be a tangent and/or cotangent function

Since each cycle/period is 2π , $B = 1/2$

The point of inflection (also, point of rotation symmetry)

is at $y = -3$ Vertical shift: down 3 $D = -3$

For tangent, there is no horizontal shift.. $C = 0$

The period is 2π , $1/4$ of the move is $\frac{\pi}{2}$

at 0, the y value is -3... Then,

at $\frac{\pi}{2}$, the y value is -2 ----- a change of +1

$$y = \tan\left(\frac{1}{2}x\right) - 3$$

Amplitude $A = 1$

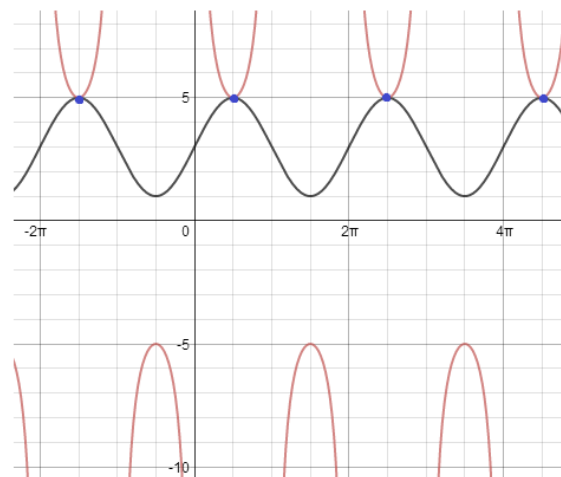
For reciprocal cotangent, the zeros become asymptotes and the asymptotes become zeros....

horizontal shift of π

Also, remember tangent and cotangent go in opposite directions....

amplitude $A = -1$

$$y = -\cot\left(\frac{1}{2}(x - \pi)\right) - 3$$



III. System of equations

Solve the following algebraically. Then, graph to confirm your solutions.

$$y = 2\sin x + 3$$

$$y = 5\csc x$$

To solve, set equations equal to each other...

$$2\sin x + 3 = 5\csc x$$

$$\left(2\sin x + 3 = 5 \frac{1}{\sin x}\right) \cdot \sin x$$

$$2\sin^2 x + 3\sin x = 5$$

$$2\sin^2 x + 3\sin x - 5 = 0$$

$$(2\sin x + 5)(\sin x - 1) = 0$$

$$\sin x - 1 = 0$$

$$x = \frac{\pi}{2} + 2\pi k$$

$$2\sin x + 5 = 0$$

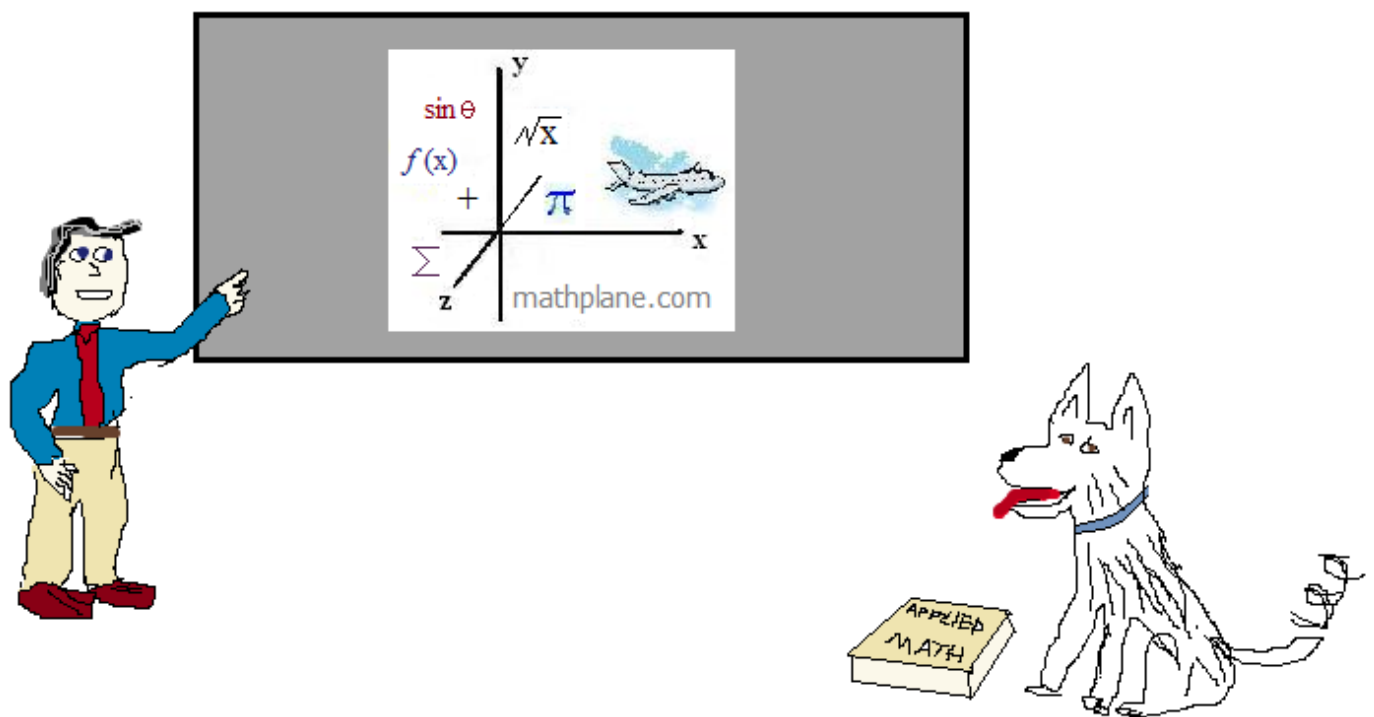
$$\sin x = -5/2 \text{ EXTRANEIOUS}$$

These are points where the graphs intersect!

Thanks for downloading this packet. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



All proceeds go to site maintenance and improvement. (Plus, treats for Oscar the Dog!)

Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest